

# Calculus III

## Test

Nov 10, 2006

Time allowed: 45 min

Name:

student number:

All questions are multiple-choice. Please enter answers by marking the appropriate box. Blank sheets are provided for rough work.

The maximum mark is 35.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks.

*There are several equivalent but not identical versions of this paper. Do not be concerned if others appear to have a different paper.*

1.  $S(x)$  is the Fourier series of the function

$$f(x) = \begin{cases} (x + 2\pi)^2 & \text{if } -\pi < x < 0 \\ x^2 & \text{if } 0 < x < \pi. \end{cases}$$

$S(0) =$

☐ 0.

☐  $2\pi^2$ .

☐  $4\pi^2$ .

2. The functions extremizing  $\int_0^{\pi/2} (y'^2 - k^2 y^2) dx$  with  $y(0) = 0$  and  $y(\pi/2) = 1$ , where  $k$  is a constant, are

☐  $y = \sin kx$ .

☐  $y = \cos kx$ .

☐  $y = \sinh kx / \sinh(k\pi/2)$ .

3. The expression  $\mathbf{r} = \cos t \mathbf{i} + \sin t \mathbf{j} + t\mathbf{k}$  for  $0 \leq t \leq \infty$  represents

☐ a general position vector.

☐ the circle of radius 1 centred at the origin in the  $(x, y)$  plane.

☐ a helix spiralling round and moving along the  $z$ -axis.

4. If  $\mathbf{F} = 2z\mathbf{i} + 3x\mathbf{j} - 4y\mathbf{k}$ , the integral  $\int \mathbf{F} \cdot d\mathbf{r}$  along the curve  $\mathbf{r} = 2 \cos t \mathbf{i} + \sin t \mathbf{j}$  from  $t = 0$  to  $t = 2\pi$  is

☐  $3\pi$ .

☐ 0.

☐  $6\pi$ .

5.  $\mathbf{F} = x^2 y \mathbf{i} + (3x - yz) \mathbf{j} + z^3 \mathbf{k}$  is a vector field. Its divergence  $\nabla \cdot \mathbf{F}$  is

☐  $2xy + x^2 + 3 - y - z + 3z^2$ .

☐  $2xy - z + 3z^2$ .

☐  $2xy \mathbf{i} - z \mathbf{j} + 3z^2 \mathbf{k}$ .

Continued overleaf

6. In the half range Fourier cosine series,  $a_0/2 + \sum_1^\infty a_n \cos nx$ , for the function  $f(x) = \sin x$  ( $0 < x < \pi$ ),

☐  $a_0 = 0$  and  $a_1 = 0$ .

☐  $a_0 = 4/\pi$  and  $a_1 = 0$ .

☐  $a_0 = 0$  and  $a_1 = 1$ .

☐  $a_0 = 4/\pi$  and  $a_1 = 1$ .

7.  $I = \int_a^b L dx$  for  $L = y'^2 + x^2 y^2$ . Which of the following is correct for any function which extremizes  $I$  subject to fixed values of  $y$  at  $a$  and  $b$ ?

1.  $y'(\partial L / \partial y') - L = \text{constant}$ .

2.  $d/dx(\partial L / \partial y') - \partial L / \partial y = 0$ .

☐ Both statements.

☐ Statement 1 only

☐ Statement 2 only

☐ Neither of the two statements.

8. Which of the following is false for the plane through the point  $\mathbf{i} + \mathbf{j} + \mathbf{k}$  and containing the vectors  $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $5\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ ?

☐  $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$  is perpendicular to the plane.

☐ The line  $\mathbf{r} = (1 + 3t)\mathbf{i} + (1 - t)\mathbf{j} + (1 + 2t)\mathbf{k}$  lies in the plane.

☐ The plane has equation  $x - y - 2z + 1 = 0$ .

☐ The point on the plane closest to the origin is at a distance  $\sqrt{3}$ .

9.  $V = x^2 z^2 + y^3 z$ . The value of  $\nabla V$  at the point  $(1, -1, 2)$  is

☐  $8\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ .

☐  $8\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$ .

☐ 17

☐ 14

10. The curl of the vector  $(x^2 - y, y^2 + x, z^2)$  is

☐  $2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ .

☐  $2\mathbf{k}$ .

☐  $2(x + y)\mathbf{i} - 2(y + z)\mathbf{j} + 2(z + x)\mathbf{k}$ .

☐  $\mathbf{0}$ .

End of test